Substructuring with Subcomponent Models Based on Nonlinear Normal Modes

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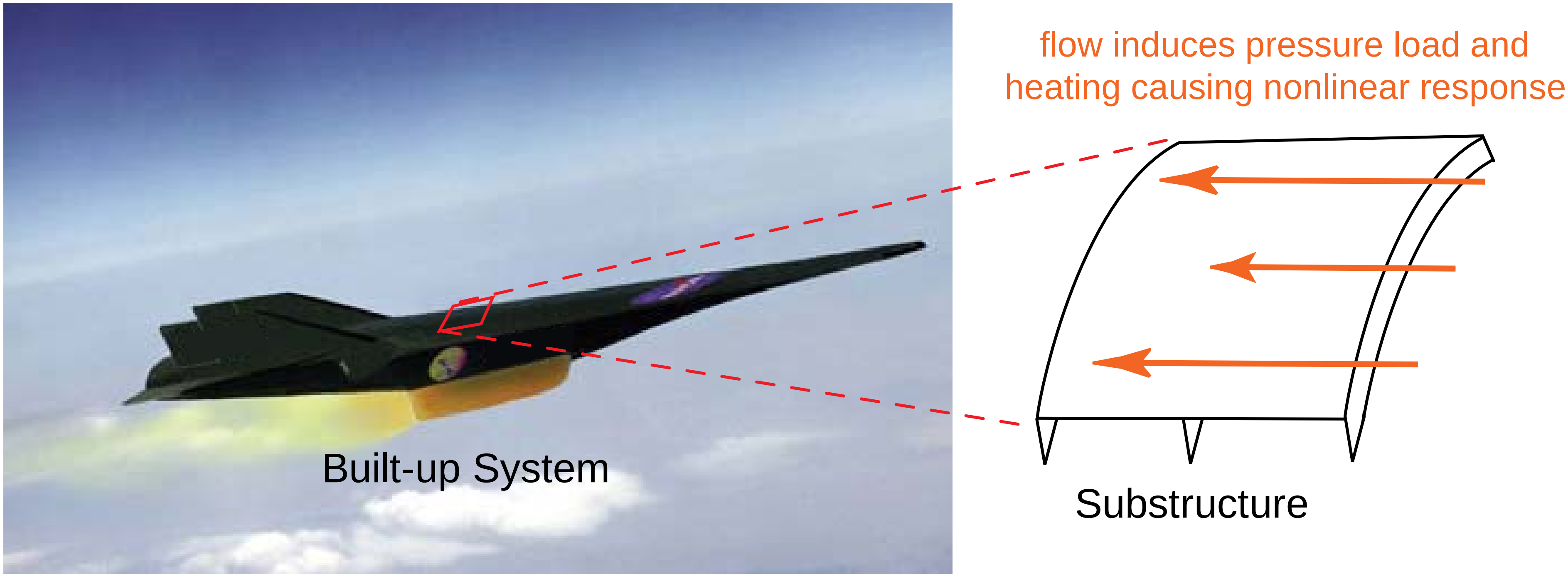
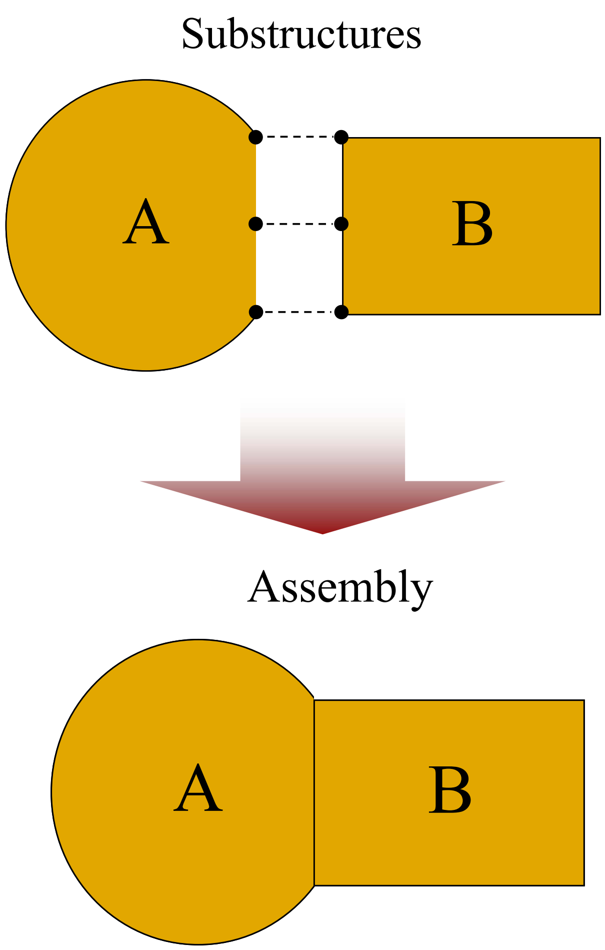
Motivation



- Structural panels on future hypersonic vehicles are subject to buckling due to thermal expansion, so their dynamic response is highly nonlinear.
- Current methods are too expensive to allow modeling of the entire aircraft, but substructures can be studied.
  - How does one nonlinear subcomponent affect the system?
  - Do concepts from linear modal substructuring transfer to nonlinear systems?

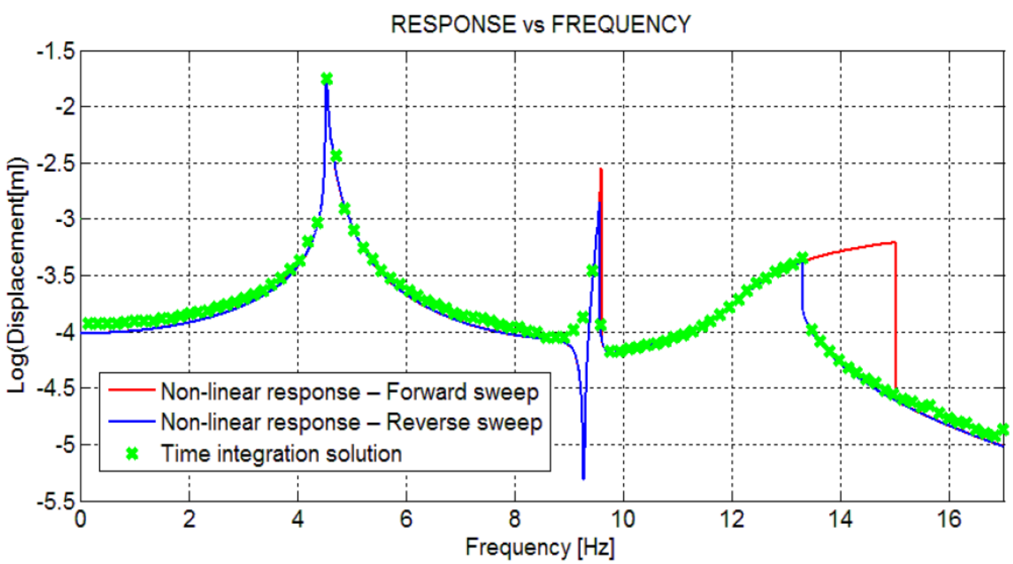
Outline

- Background
- Nonlinear normal mode (NNM) theory
  - Frequency-Energy dependence
    - Relationship to free response, forced response
  - Nonlinear Mode Shapes
- Substructuring in terms of NNMs
  - Simple example problem to illustrate the concepts.
  - Predicting NNMs of Assemblies with "Representative Linear Modal Models" (RLMMs)
- Conclusions



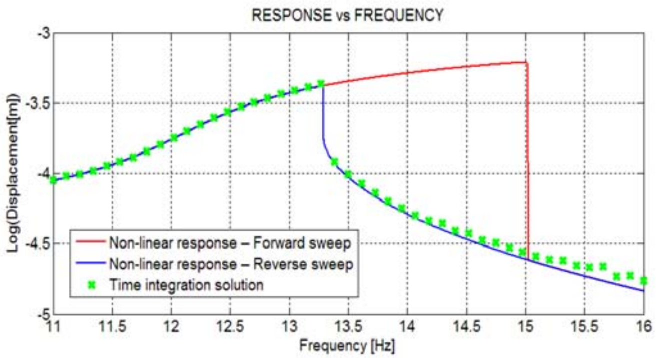
Nonlinear Assembly

- The basic idea of substructuring by satisfying compatibility and equilibrium at the interfaces between subcomponents is general and readily applicable to nonlinear substructures.
  - Commonly used in geometrically nonlinear FEA packages, for example.
  - However, this gives little insight into how the dynamics of each substructure affect the nonlinear behavior.



Prior Studies: Ozguven et al.

- Quite general method for frequency based substructuring presented in mid-90's by Watanabe and Sato [1], Comert & Ozguven [2] and Ferreira & Ewins [3].
  - Based on a single frequency **harmonic balance model (HBM**) and **describing functions.**



$$\ddot{x}+2\zeta\omega_1\dot{x}+\omega_1 x+\omega_3 x^3 = f(t) \longrightarrow x(t)=A\sin(\omega t)\longrightarrow$$

$$-\omega^2 A+2\zeta\omega_n i\omega A+\left(\omega_n{}^2+\tfrac{3}{4}\omega_3\left|A\right|^2\right)A=f(t)$$

  - Satisfies equilibrium and compatibility of the dominant harmonic in the HBM.
  - Models exist in the physical domain; no mention of modes and no method for reduction of the nonlinear model.

Quasi-linearization

Harmonic Balance

Describing Function Analysis

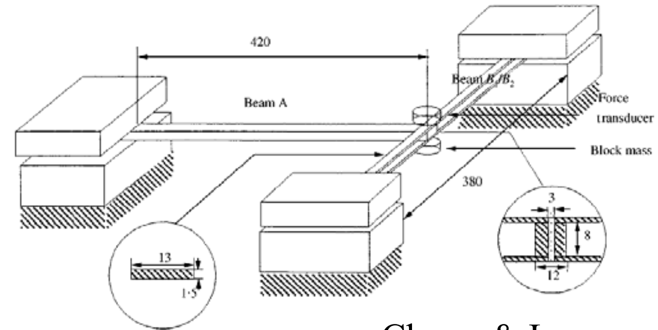
[1] K. Watanabe and H. Sato, "Development of Nonlinear Building Block Approach," Journal of Vibration, Acoustics, Stress, and Reliability in Design, vol. 110, pp. 36-41, 1988.

[2] M. D. Cömert and H. N. Ozguven, "A Method for Forced Harmonic Response of Substructures Coupled by Nonlinear Elements," in Proceedings of the 13th IMAC Nashville, Tennessee, 1995.

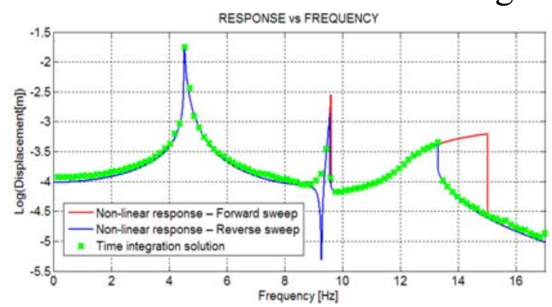
[3] J. V. Ferreira and D. J. Ewins, "Nonlinear receptance coupling approach based on describing functions," in Proceedings of 14th IMAC Dearborn, Michigan, USA, 1996.

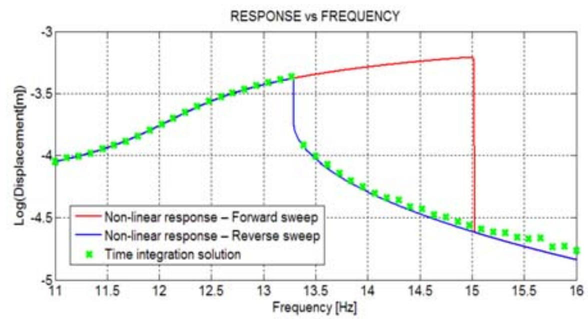
Prior Studies

- Chong & Imregun built on these concepts presenting a modal substructuring method for nonlinear systems in [4].
  - "Nonlinear Modes" defined either as eigenvalues of [M] and tangent stiffness matrix $[K(X_{max})]$ or as shape and frequency satisfying a single term HBM.
- Allen & Kuether presented a substructuring / structural modification framework in terms of Rosenberg's nonlinear normal modes.
  - Showed excellent performance for realistic problems.
  - A formal proof has yet to be developed.



Chong & Imregun





[4] Y. H. Chong and M. Imregun, "Coupling of non-linear substructures using variable modal parameters," MSSP, vol. 14, pp. 731-746, 2000.
[5] T. Kalaycioglu and H. N. Ozguven, "Harmonic response of large engineering structures with nonlinear modifications," in EURODYN Leuven, Belgium, 2011.

Modal Substructuring with Nonlinear Normal Modes

- Allen & Kuether's work explores **modal substructuring** starting with a more concrete definition for the Nonlinear Normal Modes (NNM) by Rosenberg [1] (1960)
  - This definition has been used and generalized by Vakakis [2] & Kerschen et al. [3].
  - Defined as periodic responses of the undamped nonlinear equations of motion - exact for a given model.
  - Valid even for cases of internal resonance (when single-term harmonic balance solutions become invalid)
- How do the nonlinear modes of an assembly affect the response of the nonlinear system?
  - When do insights from linear substructuring apply? (e.g. thinking of the NNMs as modes of the substructures)

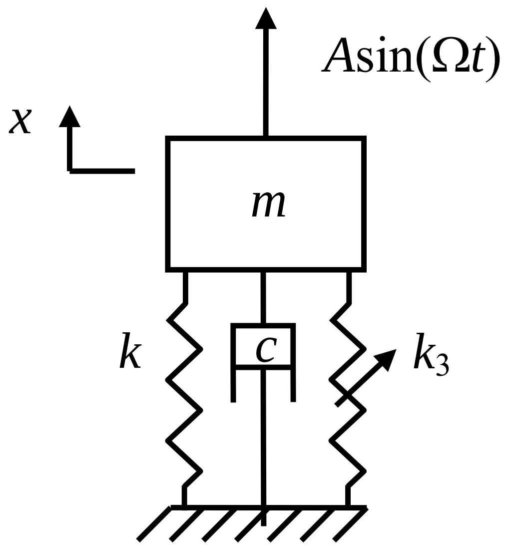
[1] R. M. Rosenberg, "Normal modes of nonlinear dual-mode systems," Journal of Applied Mechanics, vol. 27, pp. 263–268, 1960.

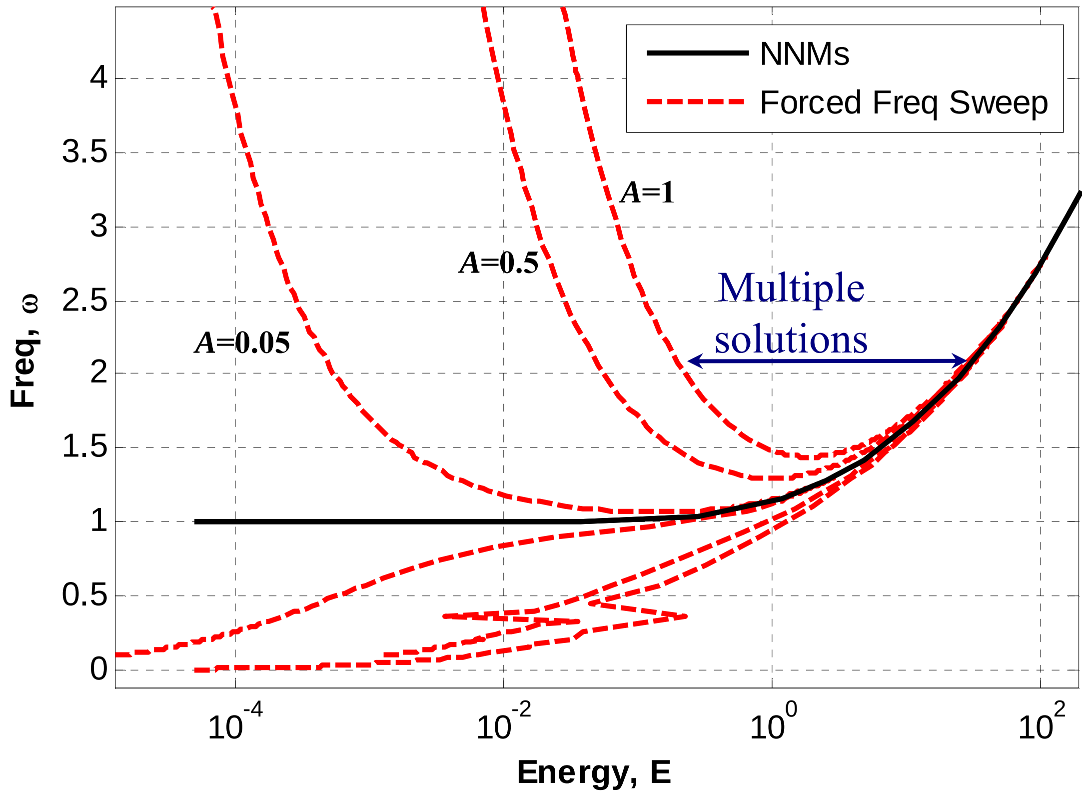
[2] A. F. Vakakis, "Non-linear normal modes (NNMs) and their applications in vibration theory: an overview," Mechanical Systems and Signal Processing, vol. 11, pp. 3-22, 1997..

[3] G. Kerschen, M. Peeters, J. C. Golinval, and A. F. Vakakis, "Nonlinear normal modes. Part I. A useful framework for the structural dynamicist," Mechanical Systems and Signal Processing, vol. 23, pp. 170-94, 2009.

Nonlinear Normal Mode (NNM) Theory

$$\ddot{x} + 2\zeta\omega_1\dot{x} + \omega_1 x + \omega_3 x^3 = f(t)$$



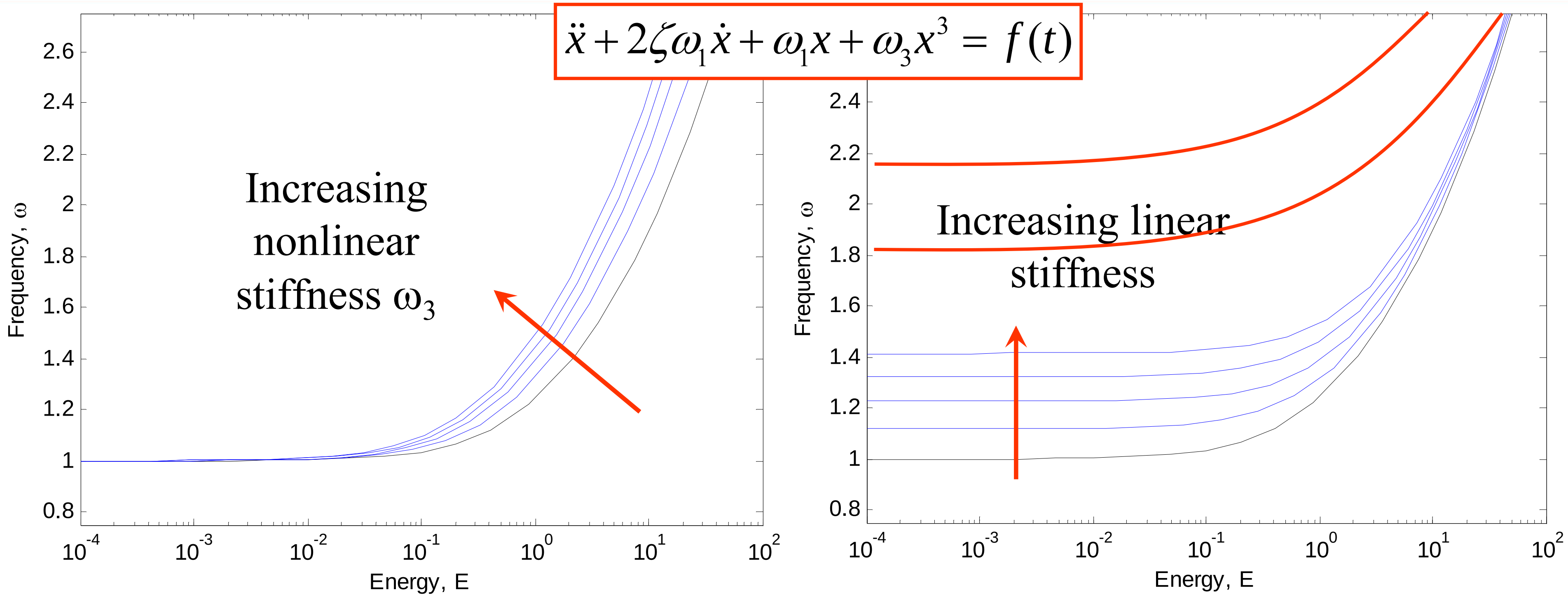


- Undamped NNM defined as a periodic response of the undamped nonlinear system.
  - Damped NNM definition also exists (Pierre & Shaw)
  - NNMs related to free response of undamped system, forced periodic response and potentially even to random response.
- **NNMs in this work were computed using algorithm in [1].**

[1] M. Peeters, R. Viguie, G. Serandour, G. Kerschen, and J. C. Golinval, "Nonlinear normal modes, part II: toward a practical computation using numerical continuation techniques," Mechanical Systems and Signal Processing, vol. 23, pp. 195-216, 2009..

Effect of System Parameters on NNM Frequency

$$\ddot{x} + 2\zeta\omega_1\dot{x} + \omega_1 x + \omega_3 x^3 = f(t)$$



- Changing the parameters of a nonlinear system causes intuitive changes in the basic structure of the NNM frequency-energy dependence.
  - Similar analyses could be performed for other nonlinearities to tailor the substructure to eliminate undesirable nonlinear behavior or accentuate desirable behavior.
  - (If internal resonance occurred, e.g. for an MDOF system, the result would be considerably more complicated.)

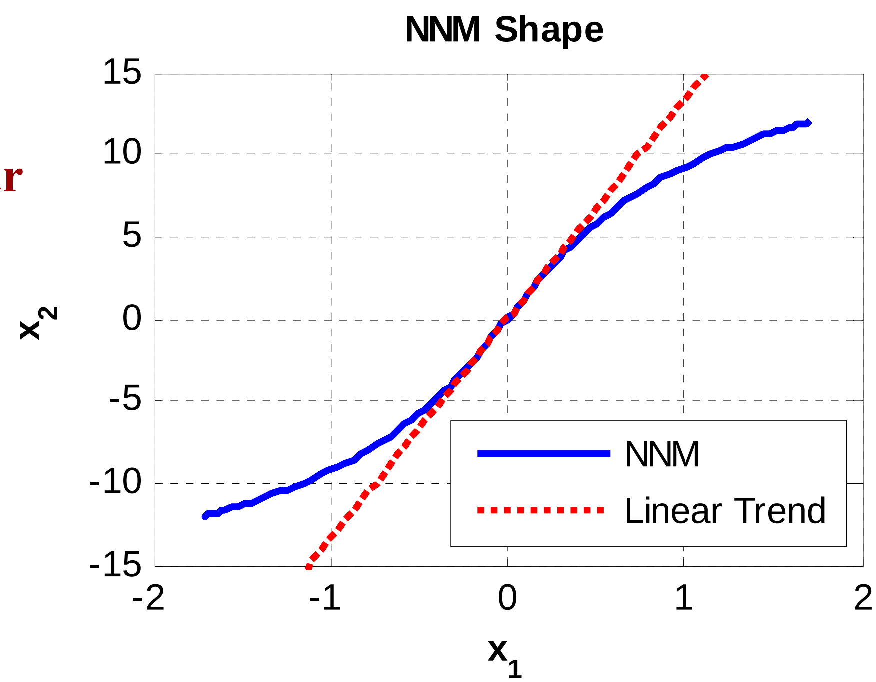
NNMs of MDOF Systems

- Multi-degree-of-freedom systems have multiple nonlinear normal modes.
- Each is characterized by its frequency-energy dependence (analogous to its natural frequency) and mode shape.
- Mode shapes of NNMs are nonlinear functions of the modal amplitude.

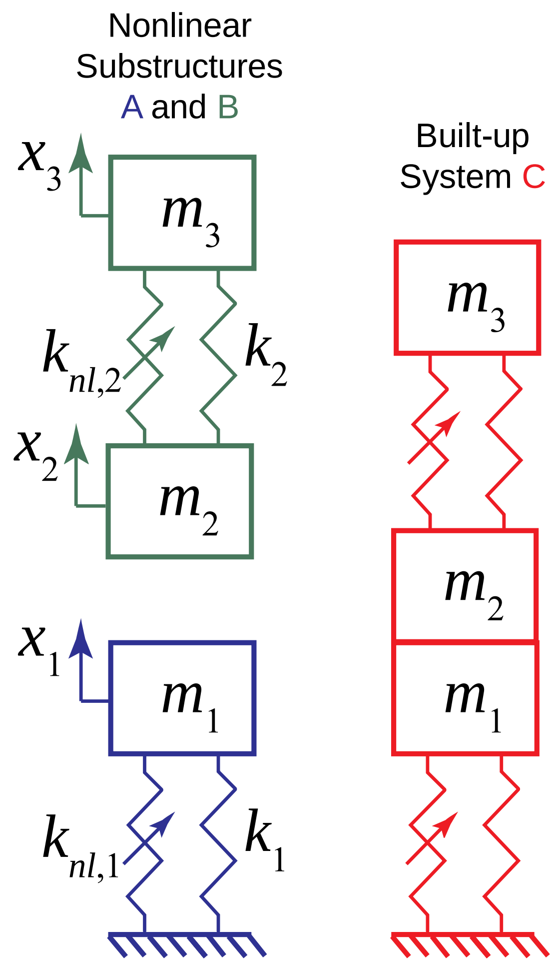
$$\mathbf{x} = \boldsymbol{\varphi}_r(q_r) \quad \textbf{Nonlinear}$$

$$\mathbf{x} = \begin{bmatrix} \phi_{1r} & \phi_{2r} & \cdots & \phi_{Nr} \end{bmatrix}^{\mathrm{T}} q_r \quad \textbf{Linear}$$

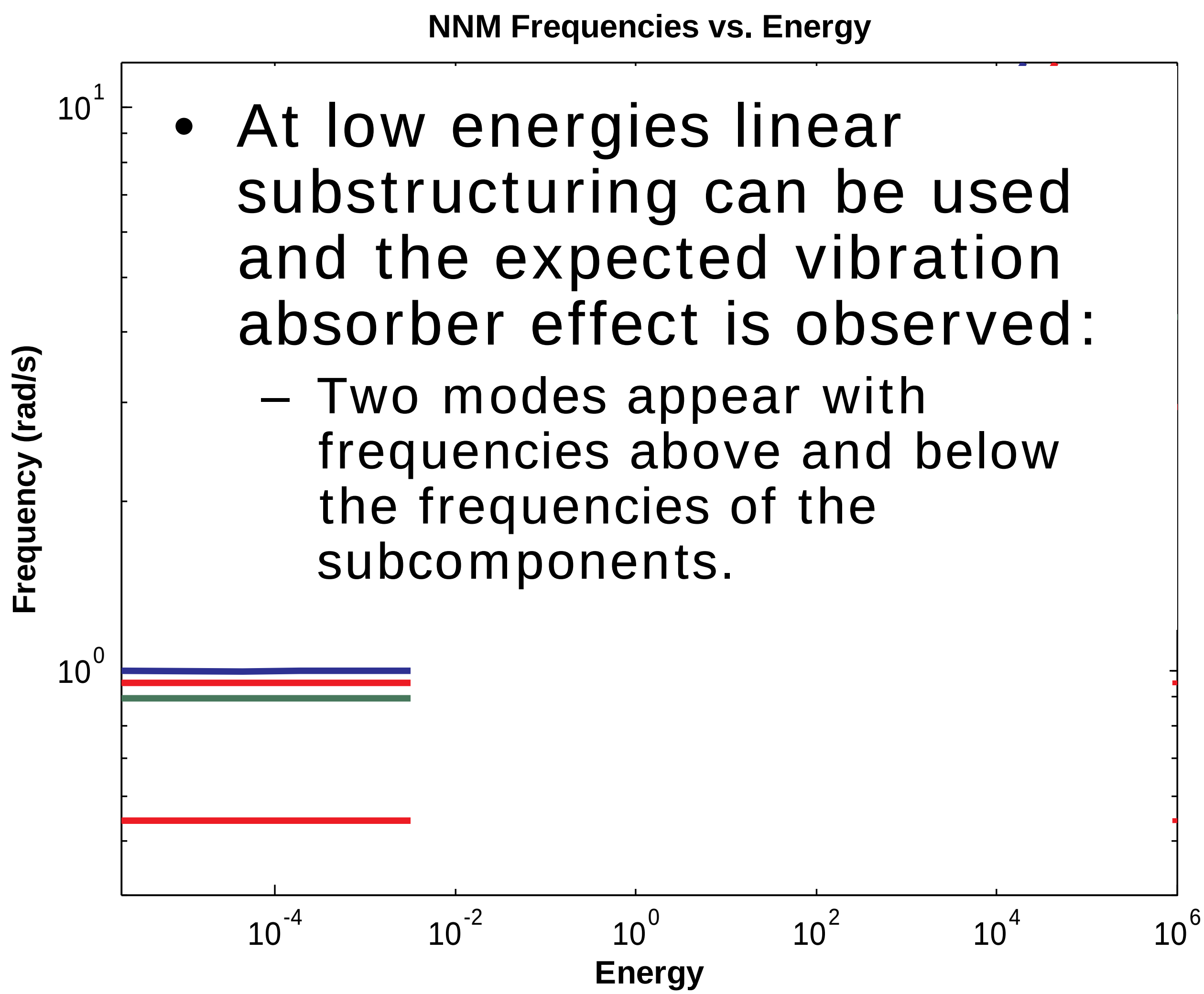
- The NNMs are related to a manifold in the state space.
- Motions that initiate on an NNM manifold remain on that manifold for all time, described by a single-degree-of-freedom equation of motion.



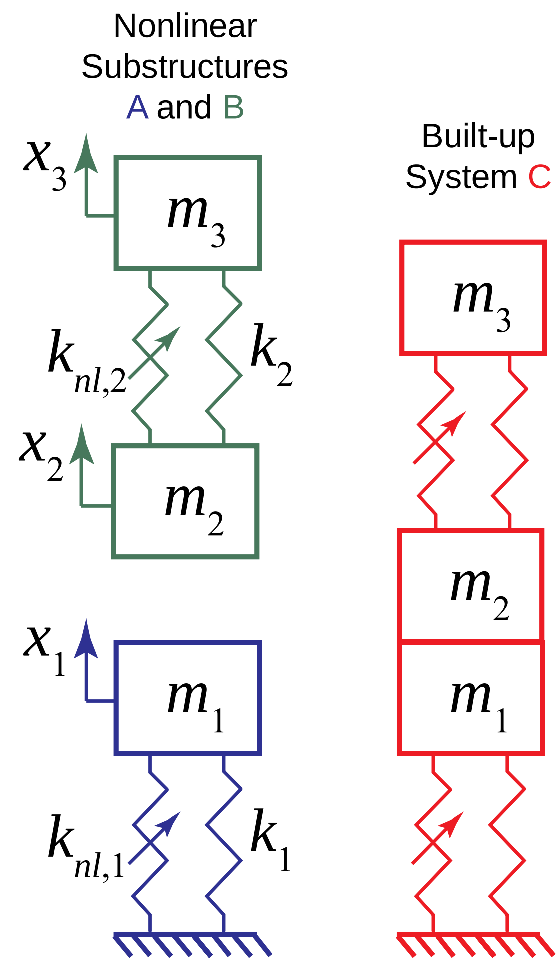
- Example: Two simple nonlinear systems joined to form a built-up structure.
  - Parameters:
    - $m_1=1$, $m_2=0.5$, $m_3=0.5$, $k_1=1$, $k_2=0.2$
  - Nonlinear springs are cubic with coefficients $k_{\mathrm{nl},1}=0.5$ and $k_{\mathrm{nl},2}=1\mathrm{e}\text{-}5$.
  - Linearized natural frequency of A, $\omega_{A1}=1$ rad/s, is close to the second linearized natural frequency, $\omega_{B2}=(2k_2/m_2)(1/2)\approx 0.89$.
    - Attached system acts as a vibration absorber.
    - Using linear theory, one would expect that the natural frequencies of the built up system will be above and below $\omega_{A1}\approx\omega_{B2}$.

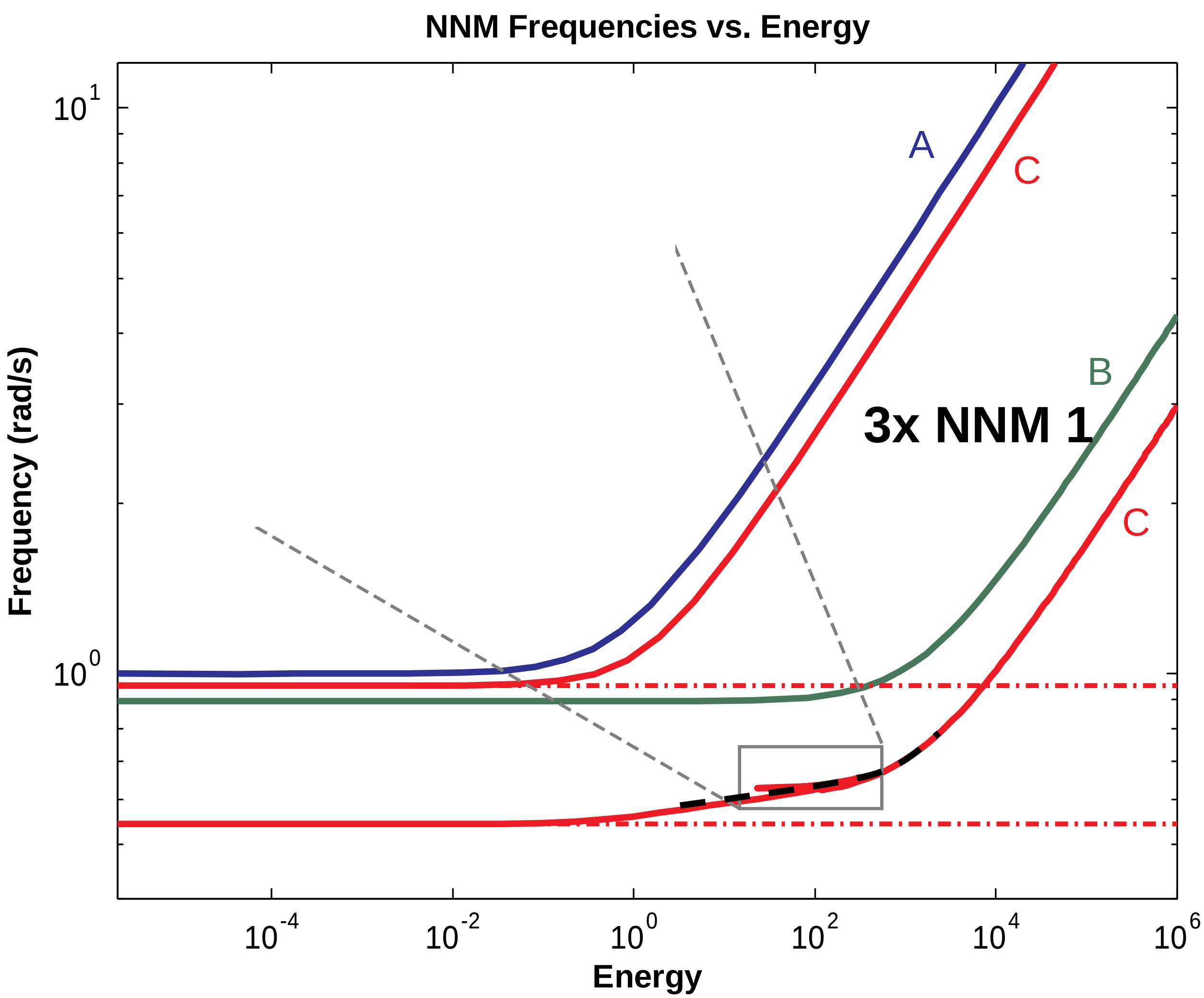


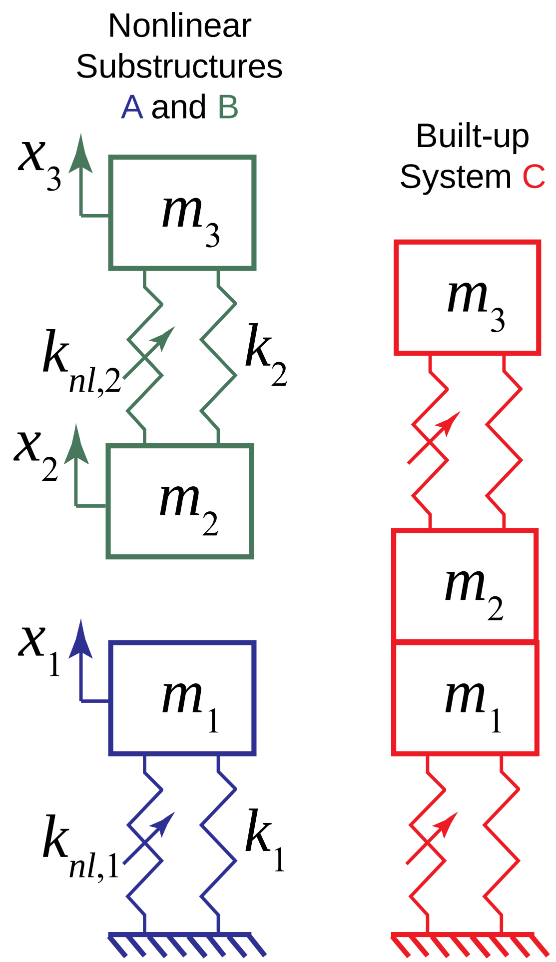
Substructuring in light of Nonlinear Normal Modes



- At low energies linear substructuring can be used and the expected vibration absorber effect is observed:
  - Two modes appear with frequencies above and below the frequencies of the subcomponents.



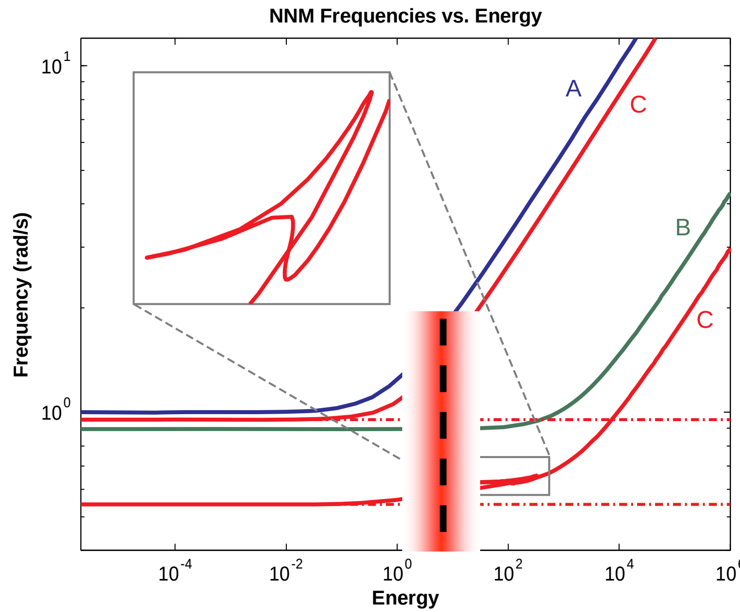




- Behavior resembles linear substructuring behavior for all but one small range of energy where internal resonance occurs.
- **Can the evolution be understood using linear substructuring concepts?**

Quasi-Linear Substructuring

- Linear substructuring:
  - Combines the modes of two substructures (using constraints and equilibrium) to estimate the modes of an assembly.
  - e.g. "**`ritzscomb.m`**" package by Allen
- Can the NNMs of two substructures be combined in a similar way to estimate the NNMs of the assembled system at a certain energy?



Modes of Substructures:

$$\left(\omega_r\right)^{\mathrm{A}} \quad \left(\phi_r\right)^{\mathrm{A}}$$

$$\left(\omega_r\right)^{\mathrm{B}} \quad \left(\phi_r\right)^{\mathrm{B}}$$

$$\cdots$$

Constraints:

$$\left[\mathbf{a}_{\mathrm{p}}\right]\begin{Bmatrix}\mathbf{y}_{\mathrm{A}} \\ \mathbf{y}_{\mathrm{B}} \\ \vdots\end{Bmatrix} = 0$$

Linear Modal Substructuring

Modes of Assembly:

$$\omega_r \quad \phi_r$$

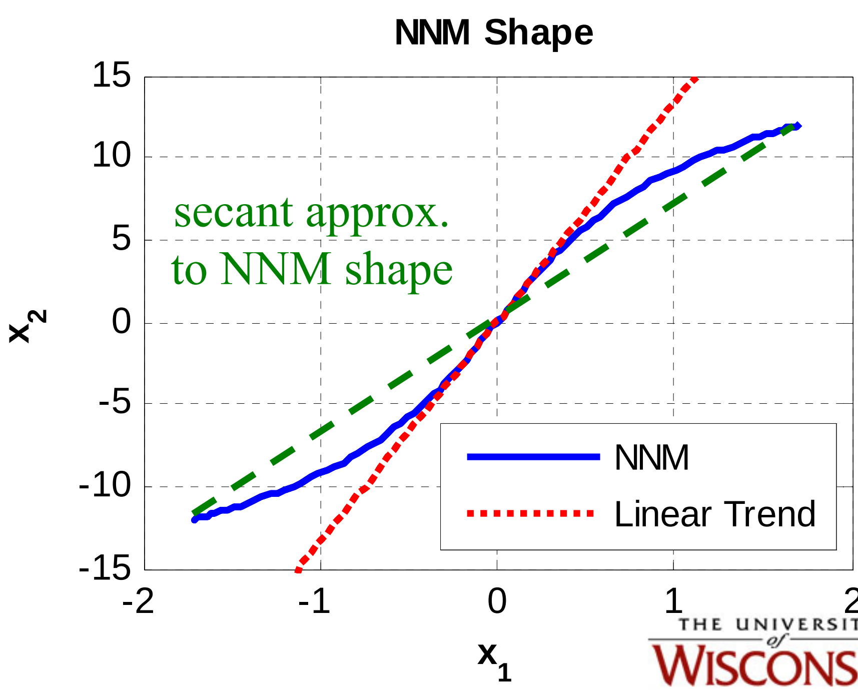
Iterate to use subcomponent modal parameters at the correct energy level.

- Proposed Approach:
  - Define a linear model that captures the frequency and mode shape of each mode at a certain energy level:
    - Representative Linear Modal Model (RLMM).
    - Quasi-linear modal model
  - Use linear substructuring to couple the RLMMs and estimate the NNMs of the assembly at that energy level.

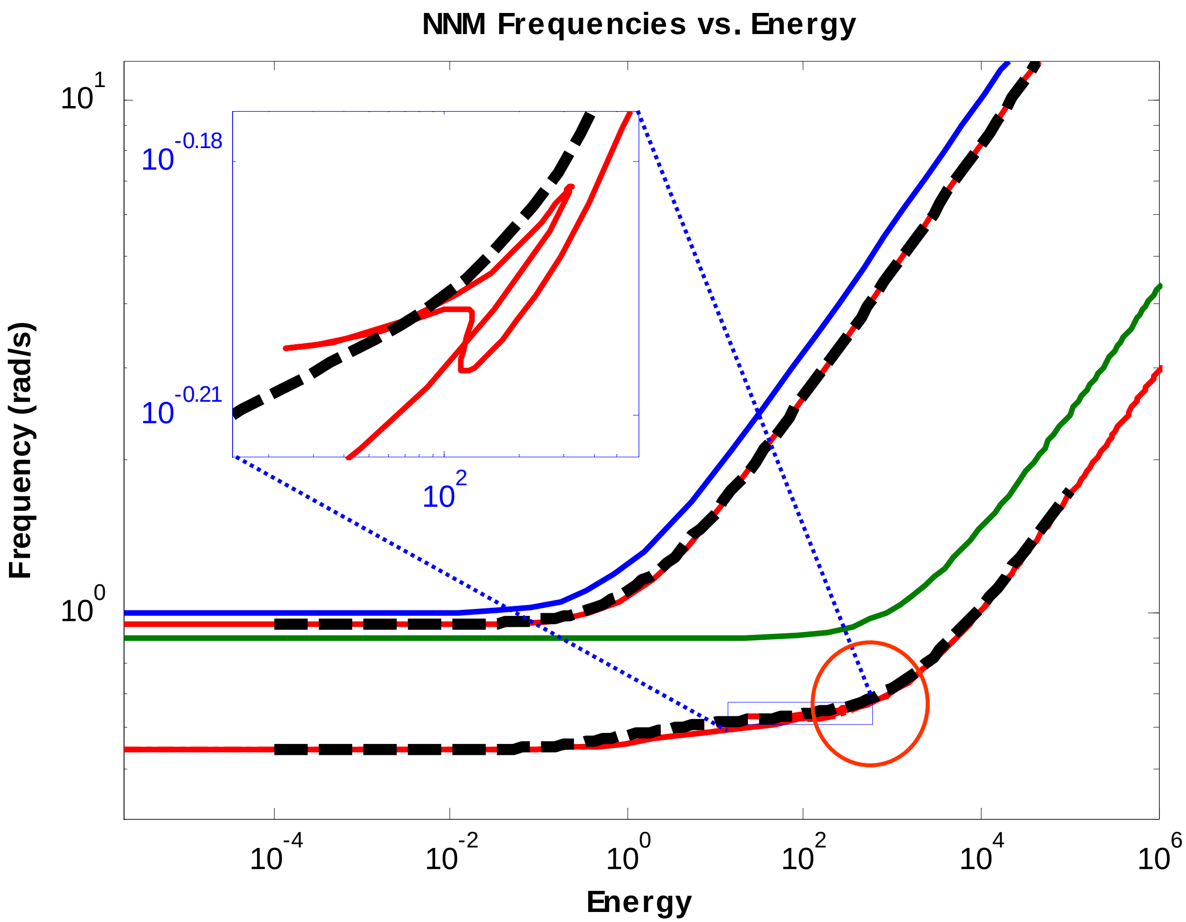
$$\boxed{\omega^{\mathrm{R}}(E)_r} = \omega^{\mathrm{NNM}}(E)_r$$

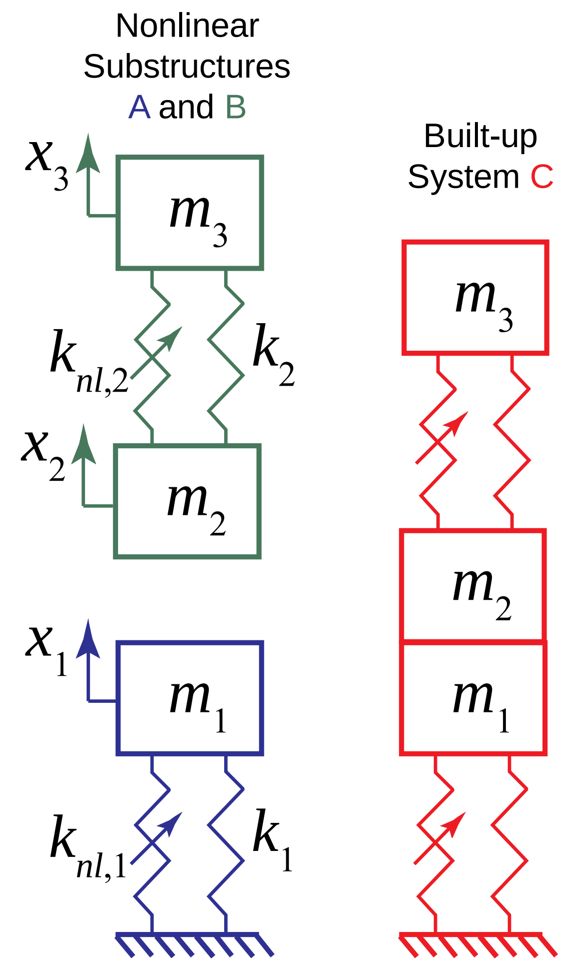
$$\boxed{\phi^{\mathrm{R}}(E)_r} = \boldsymbol{\varphi}^{\mathrm{NNM}}(E, t_{\mathrm{dpk}})_r$$

RLMM



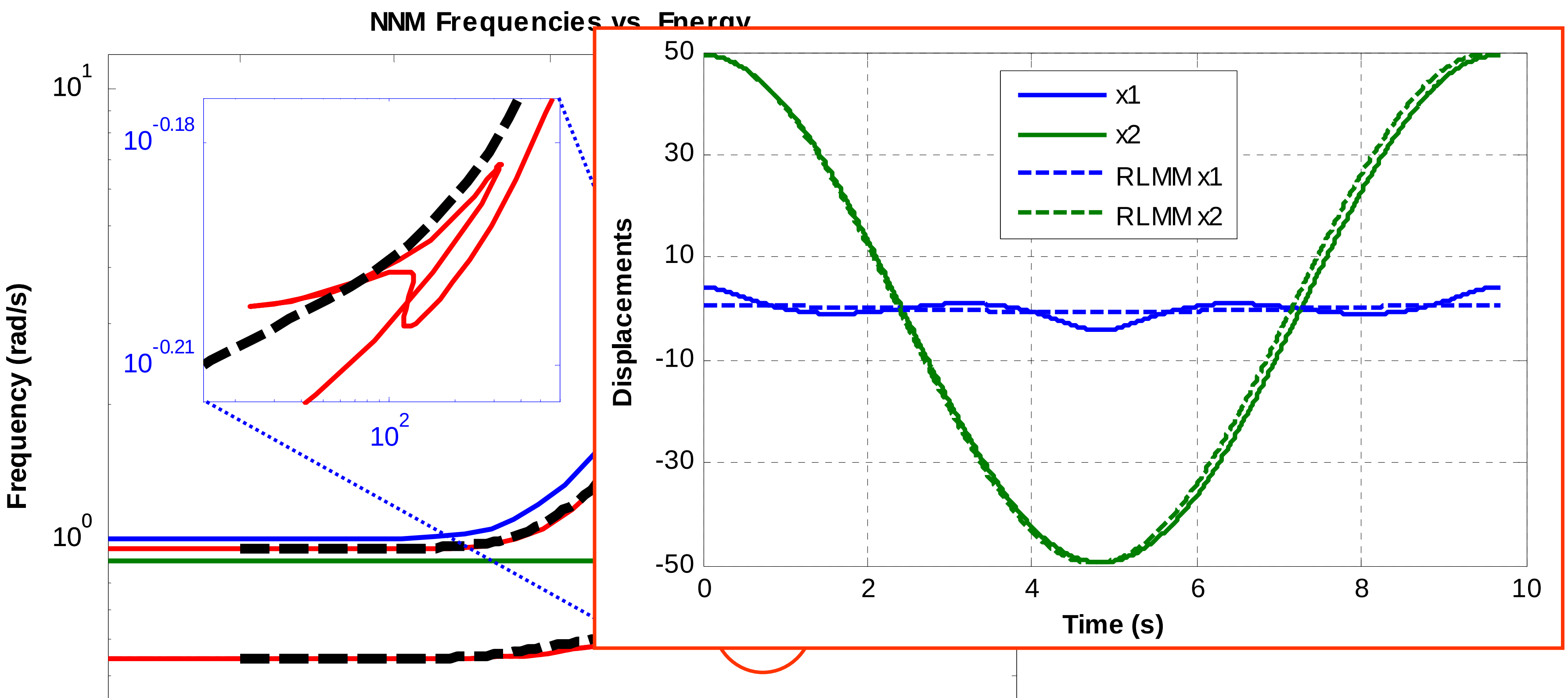
RLMM Predictions for Simple Spring-Mass System





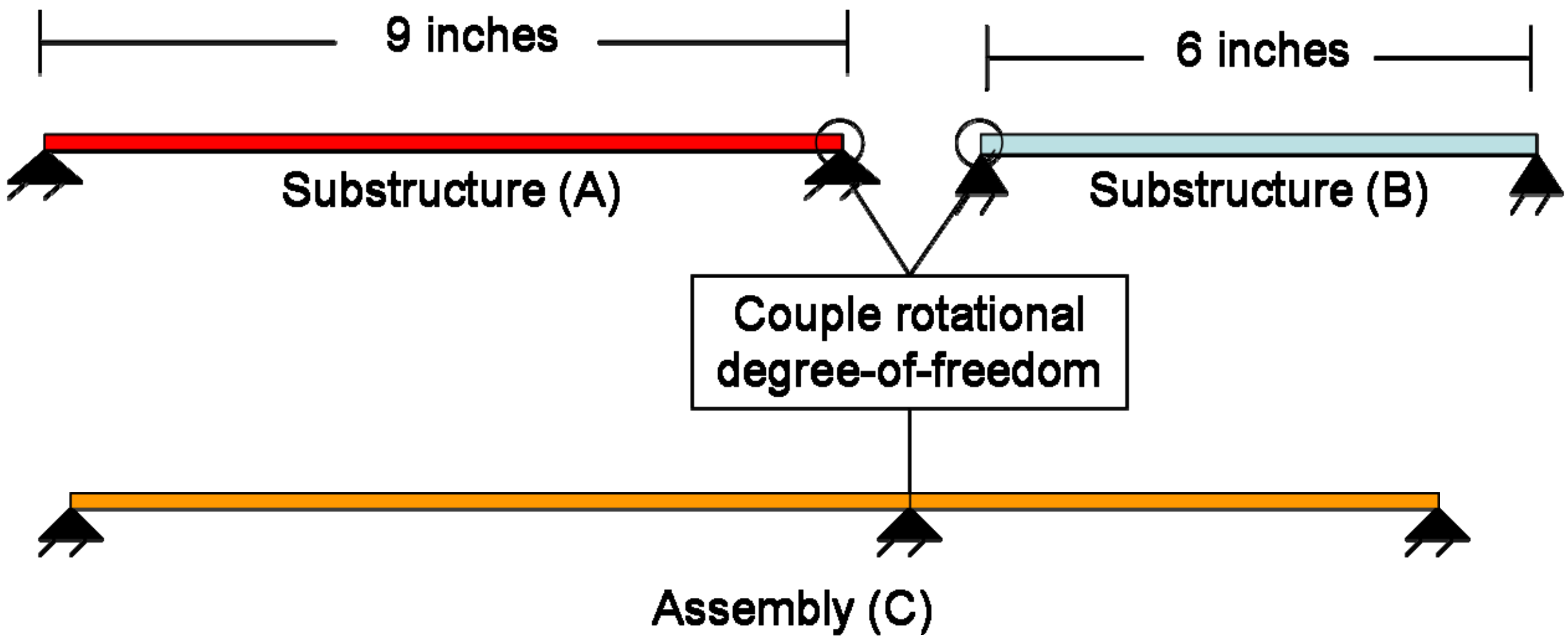
- RLMM approach predicts the NNM frequency-energy dependence very precisely for this system!
  - Internal resonance is not captured but one could ascertain that it is possible due to the 3:1 ratio of NNM frequencies.

RLMM Predictions for Simple Spring-Mass System

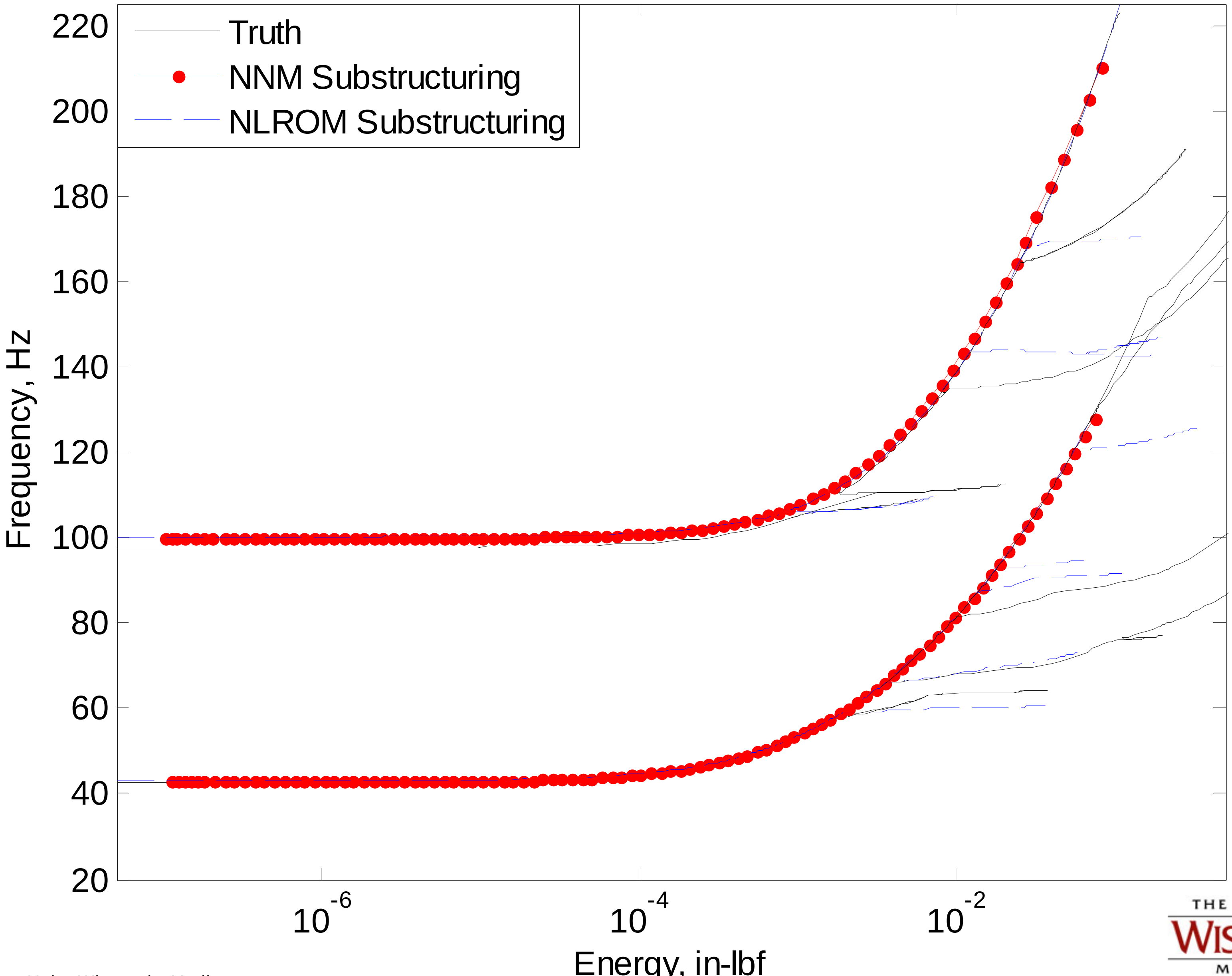


- NNM is simply a periodic solution to the governing differential equations.
- As a linear model, the RLMM approximates the NNM as a pure cosine.
  - Agreement is quite good in this case.

- Modal reduction at the subcomponent level (compared to 119 DOF and 89 DOF).
  - QL-NNM Substructuring: Each beam has 8 NNMs.
  - NLROM Substructuring: NLROMs constructed from 8 linear modes.



Results of Nonlinear Modal Substructuring



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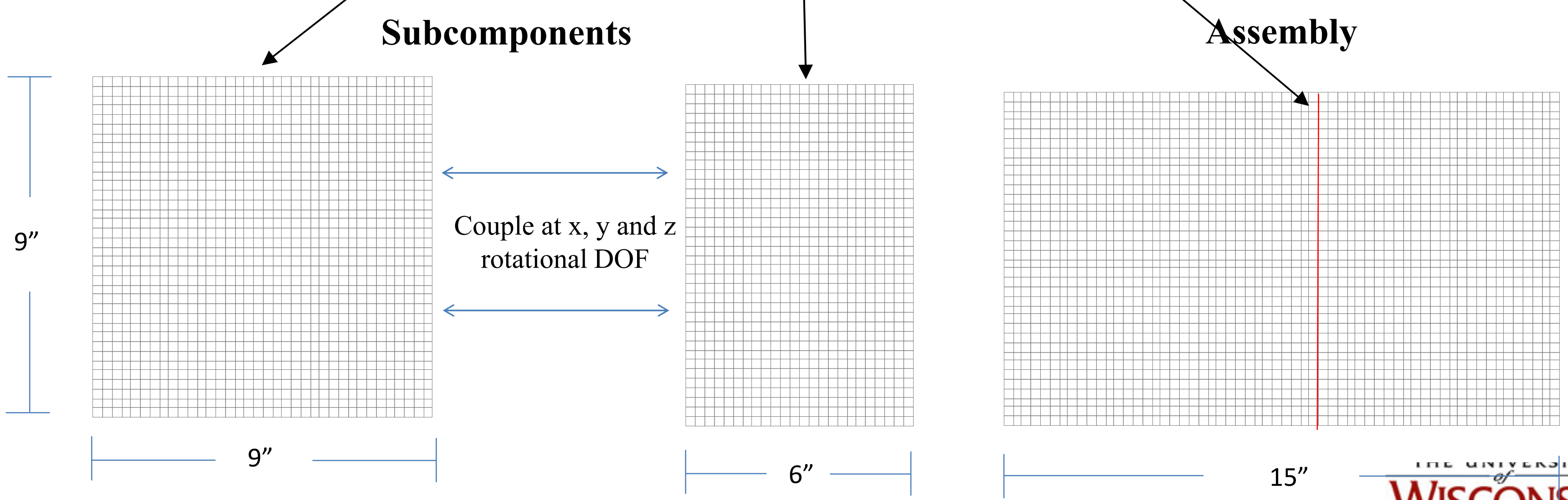
Further Extensions by Allen & Kuether

1. M. S. Allen and R. J. Kuether, "Substructuring with Nonlinear Subcomponents: A Nonlinear Normal Mode Perspective," in *30th International Modal Analysis Conference* Jacksonville, Florida, 2012
2. R. J. Kuether and M. S. Allen, "Structural Modification of Nonlinear FEA Subcomponents Using Nonlinear Normal Modes," in *31st International Modal Analysis Conference (IMAC XXXI)* Garden Grove, CA, 2013
3. R. J. Kuether and M. S. Allen, "Nonlinear Modal Substructuring of Systems with Geometric Nonlinearities " in *54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference* Boston, MA, 2013
4. R. J. Kuether and M. S. Allen, "Substructuring with Nonlinear Reduced Order Models and Interface Reduction with Characteristic Constraint Modes," in *55th AIAA/ASMe/ASCE/AHS/SC Structures, Structural Dynamics, and Materials Conference* National Harbor, Maryland, 2014

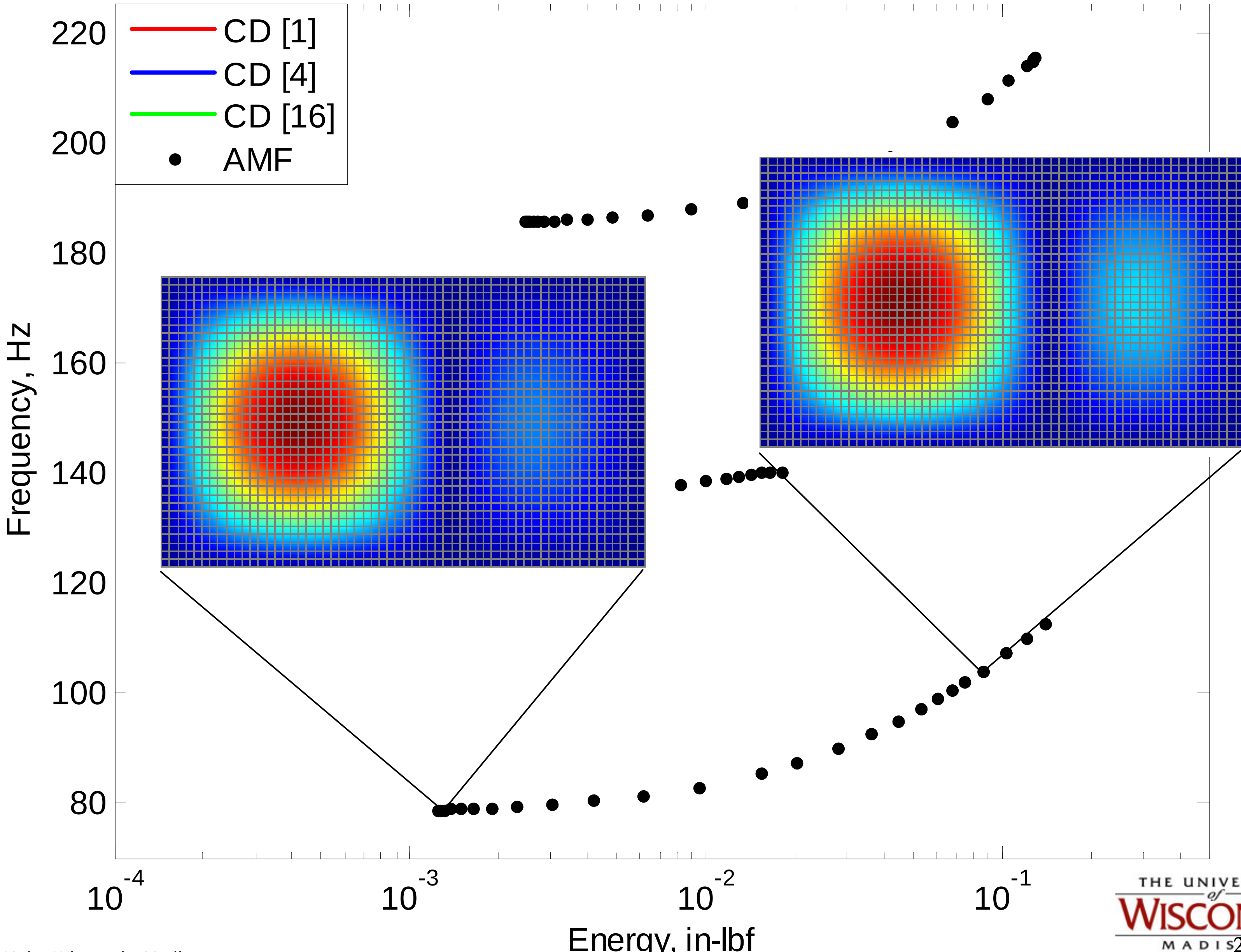
Case Study: Assembly of two flat plates

- *Subcomponent A*: 9”x9”x0.031” plate
- *Subcomponent B*: 9”x6”x0.031” plate
- Material properties of structural steel

Each plate has simple supports at edges



Nonlinear Substructuring Results



Nonlinear Substructuring Results